# Audio Signal Processing : VI. Denoising

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**Goal** : Removing a bakground (stationnary) noise

**Framework**

$$Y[n] = X[n] + W[n]$$

where

- $X[n]$ : audio signal (stochastic process)
- $W[n]$ : Stationnary noise stochastic proces
- $W$ and $X$ are decorrelated

$$Y[n] = X[n] + W[n], \quad \text{with } X \text{ and } W \text{ decorrelated}$$

**Framework of Wiener filtering**

Find the optimal filter $h$ such that

$$e = E((X[n] - h \star Y[n])^2)$$

is minimum

$$Y[n] = X[n] + W[n], \quad \text{with } X \text{ and } W \text{ decorrelated}$$

**Theorem** :

- Optimal filter

$$\hat{h}(e^{i\omega}) = \frac{\hat{R}_X(e^{i\omega})}{\hat{R}_X(e^{i\omega}) + \hat{R}_W(e^{i\omega})}$$

- Optimal error

$$e = E((X[n] - h \star Y[n])^2) = \frac{1}{2\pi}\int_0^{2\pi} \frac{\hat{R}_X(e^{i\omega})\hat{R}_W(e^{i\omega})}{\hat{R}_X(e^{i\omega}) + \hat{R}_W(e^{i\omega})} d\omega$$

In practice ?

**Goal** : Removing ($\simeq$ 1ms) "clicks" in an audio signal

**Framework** : a click is modeled by a Dirac

$$s[n] = X[n] + d_0\delta[n - n_0]$$

where

- $X[n]$ : audio signal (stochastic process)
- $\delta[n]$ : the click (a dirac)
- $d_0$ : the amplitude of the click
- $n_0$ : the position of the click

$$s[n] = X[n] + f[n - n_0]$$

**Framework of adaptive filtering**

Knowing $f[n]$

- General framework Reverse the paradigm
  - $X[n]$ : the noise
  - $f[n]$ : the signal
- find the optimal filter $h$ that maximizes the SNR at time $n = n_0$

$$\rho = \frac{\text{Energy of } h \star f[n - n_0] \text{ at } n = n_0}{\text{Energy of } h \star X[n] \text{ at } n = n_0}$$

- Threshold

$$s[n] = X[n] + f[n - n_0]$$

**Framework of adaptive filtering**

The SNR at time $n = n_0$

$$\rho = \frac{\text{Energy of } h \star f[n - n_0] \text{ at } n = n_0}{\text{Energy of } h \star X[n] \text{ at } n = n_0} \quad (1)$$

$$= \frac{(f \star h[0])^2}{E((h \star X[0])^2)} \quad (2)$$

$$s[n] = X[n] + f[n - n_0]$$

**Theorem**

The SNR at time $n = n_0$

$$\rho = \frac{(f \star h[0])^2}{E((h \star X[0])^2)}$$

is maximized for

$$\hat{h}(e^{i\omega}) = C\frac{\hat{f}(e^{i\omega})^*}{\hat{R}_X(e^{i\omega})}$$

and one gets

$$\rho\text{max} = \frac{1}{2\pi}\int_0^{2\pi} \frac{|\hat{f}(e^{i\omega})|^2}{\hat{R}_X(e^{i\omega})} d\omega$$